Flexible Content Placement in Cache Networks using Reinforced Counters

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Abstract. *In this paper we study the problem of content placement in a cache network. We consider a network where routing of requests is based on random walks. Content placement is done using a novel mechanism referred to as "reinforced counters". To each content we associate a counter, which is incremented every time the content is requested, and which is decremented at a fixed rate. We model and analyze this mechanism, tuning its parameters so as to achieve desired performance goals for a single cache or for a cache network. We also show that the optimal static content placement, without reinforced counters, is NP hard under different design goals.*

1. Introduction

In today's Internet the demand for multimedia files and the sizes of these files are steadily increasing. The popularity of Youtube, Dropbox and Netflix, to name a few, motivate researchers to seek for novel cost-effective content dissemination solutions.

Caching is one of the most classical solutions to increase the load supported by computer systems. In essence, caching consists of transparently storing data in a way that future requests can be served faster. Caching in the realm of standalone computer architectures received significant attention from the research community since the early sixties, and web-caching has also been studied for at least one decade. The objects of study of this work, in contrast, are cache networks, which were proposed and started to receive focus much more recently [Zhang and Carofiglio 2012, Jacobson et al. 2009].

Cache networks comprise two main features, caching and routing, both performed by the same core component: a cache router. In a cache network, content traverses the network from sources to destinations, but can in turn be stored in caches strategically placed on top of the routers. The *data plane* is responsible for transmitting contents whereas the *control plane* is responsible for the routing of requests. The system considered in this paper is illustrated in Figure 1. A request for file F is routed using random walks through the caches. Every time the request hits a cache, the cache selects uniformly at random one of the links that are incident to the cache and uses it to forward the request. Once the request reaches a cache where the content is stored, the content is transferred to the requester through the data plane. We assume that all contents are stored in at least one cache.

In this paper we study the problem of content placement in cache networks. We pose the following questions: (1) How to store and evict contents from a cache in a flexible and scalable manner? (2) How to efficiently and distributedly place content in a cache network?

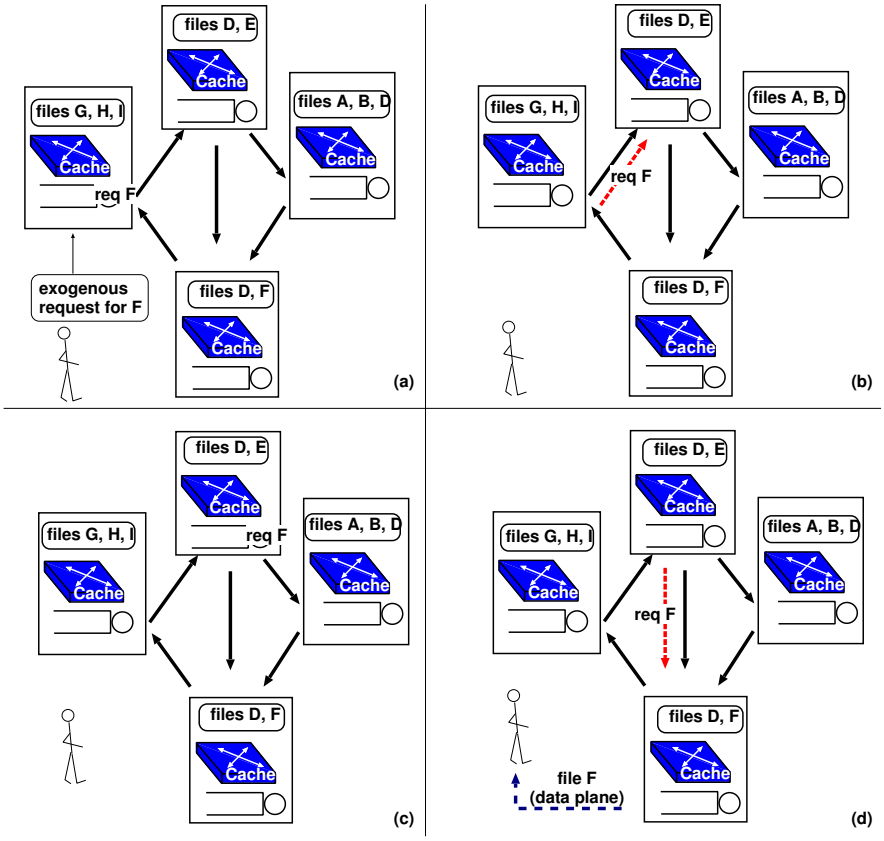


Figura 1. Illustration of how system works: (a) **a request is placed in cache 1;** (b) **the request is routed to cache 2;** (c) **the request is processed in cache 2;** (d) **the request is routed to cache 4. The file is transferred through the data plane.**

Content placement is done using a novel mechanism referred to as "reinforced counters" [Domingues et al. 2013, Domingues et al. 2014]. In [Domingues et al. 2013, Domingues et al. 2014] we have introduced RCs as an analytically-tractable solution for content placement. In this paper, we present a detailed analysis of the solution, and extend with features such as hysteresis. We then show how to use RCs to control different metrics that affect overall system performance. To each content we associate a counter, which is incremented every time the content is requested, and which is decremented at a fixed rate. We model and analyze this mechanism, tuning its parameters so as to achieve desired performance goals for both a single cache and for a cache network.

Each cache in a cache network has a given service capacity. The service capacity of a cache is related to the time it takes to 1) find content in the cache, 2) return that content to the user, through the data plane, in case of a cache hit and 3) route the request to another cache, through the control plane, in case of a cache miss. Note that in the two latter cases, the service capacity of a cache network accounts for network delays due to transmission and queueing. A cache is stable if, for a given workload, the queue of pending requests does not grow unboundedly with respect to time. A cache network is stable if, for a given workload, all its caches are stable. We show that the optimal static content placement, without reinforced counters, is NP hard under different design goals accounting for stability and content availability.

To summarize, in (partially) answering the questions above we make the following contributions

Analytical model of RC: we introduce reinforced counters as a way to flexibly store and evict contents from a cache, showing that they are amenable to analytical study and optimal tuning;

Optimal content placement: we propose a new formulation of the optimal content placement in cache networks accounting for stability. Then, we show that static content placement, without reinforced counters, is NP hard, which motivates the use of reinforced counters or variants in a network setting.

The rest of this paper is organized as follows. Section §2 studies the single cache police based on the reinforced counters. In §3 we address the problem of multiple caches and we show that under different setting the problem at hand is NP hard. In §4 we present related work, and §5 concludes.

2. Single Cache: Reinforced Counters and Flexible Content Placement

In this section, we study placement and eviction policies for a single cache under the assumption that the dynamics of each content is decoupled from the others. Decoupled content dynamics yields tractable analysis and can be used to approximate the performance of systems with fixed capacity. They are also of interest in the context of DNS caches and the novel Amazon ElastiCache system (`http://aws.amazon.com/elasticache/`).

In the policies to be introduced in this section, the expected number of items in the system can be controlled. Let π_{up} be the probability that each content is stored in the cache. Given a collection of N contents the expected number of stored contents is $N\pi_{up}$, which can be controlled or bounded according to users needs. In what follows, we consider the problem of controlling π_{up} using *reinforced counters* (Section 2.1). In Section 2.2 we illustrate how the mechanism works through some simple numerical examples. Then, in Section 2.3 we extend the reinforced counters to allow for hysteresis, showing the benefits of hysteresis for higher predictability and reduced chances of content removal before full download.

2.1. Reinforced counter with a single threshold

To each content we associate a reinforced counter [Domingues et al. 2013, Domingues et al. 2014], which is increased by one every time the content is requested, and is decreased by one as a timer ticks. Let K be the reinforced counter eviction threshold. When the counter is incremented from K to $K+1$, the content is stored. Whenever the counter is decremented from $K+1$ to K the content is evicted. The timer ticks every $1/\mu$ seconds. Henceforth, we assume that the time between ticks is exponentially distributed. This mechanism is similar to TTL caches, employed by DNS and web-caching systems [Berger et al. 2014, Fofack et al. 2012], noting that content is inserted in the cache only after the threshold K is surpassed. The advantages of using RCs over existing mechanisms will be shown in this section.

We assume that the behavior of each of the contents is decoupled. This assumption yields an analytically-tractable model and is of interest in a number of settings, including a cloud where resources are virtually unlimited.

In the cloud, we may assume that storage is infinite. Users might incur costs which are proportional to the space used, but the storage space itself is unlimited. Under

the mean field approximation [Fofack et al. 2012], it has been shown that decoupling the dynamics of multiple contents can lead to reasonable approximations to the content hit probabilities. Instead of considering that the cache has a fixed capacity, it is assumed that there are constraints on the expected number of items in the cache.

Note that under the mean field approximation the cache capacity design problem can be compared to the problem of calculating the capacity of communication lines in a telephone network. When capacity is finite, one can take advantage of statistical multiplexing either by evicting items from the cache when (a) an overflow occurs or (b) when the item expire. This is particularly relevant in the context of time-to-live caches (such as those used by the DNS system) where entries need to be renewed from time to time to avoid staleness.

Our goal in the remainder of this section is to show the advantages of having a content placement mechanism with two associated knobs, μ and K, allowing for the user to fine tune both the fraction of time in which the content is in the cache (steady state metric) while at the same time controlling the mean time between content insertions (or, equivalently, controlling the rate at which content is evicted or brought back into cache) so as to avoid that content is replaced too fast and content starvation (that is, content is never included into cache or removed from it during a finite but large time interval). The timer tick rate μ and the reinforced counter threshold K can be tuned so as to adjust the long term fraction of time in which the content is stored in the cache and to guarantee that the mean time between content eviction and content reinsertion into the cache is bounded.

In what follows, we assume that requests for a given content arrive according to a Poisson process with rate λ. λ is also referred to as the content popularity. (Recall that the behavior of each content is decoupled for others.) Figure 2 is useful to illustrate the different intervals of the cache content replacement and the notation we use. The blue (red) intervals in the figure indicate that the content is stored (or not) in cache. If content is not in cache it is brought into cache when a new request for it arrives and the reinforced counter is at the threshold K. On the other hand, if the value of the reinforced counter is at $K + 1$ and the counter ticks, the content is removed from cache. Note that we adopt the same assumption as in [Fofack et al. 2012]: the insertion and eviction of a content is not influenced by other contents in the same cache. As mentioned above, fixed storage in cache is modeled by considering the expected number of contents into the cache.

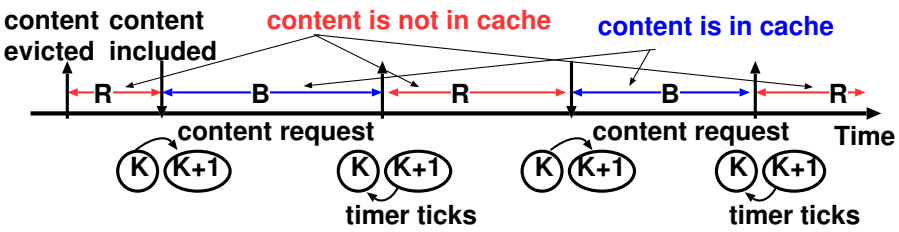


Figura 2. Request counter and notation.

Let π_{up} be the fraction of time in which the content is in the cache. Each cache content alternates from periods of inclusion and exclusion from cache (see Figure 2), and

we have (renewal theory),

$$\pi_{up} = \frac{E[B]}{E[R] + E[B]} \tag{1}$$

where $E[R]$ is the mean time that a content takes to return to the cache once it is evicted and $E[B]$ is the mean time that the content remains in the cache after insertion. It should be clear that the reinforced counter mechanism can be modeled as an M/M/1 queueing system, with state equal to the value of the reinforced counter. Then,

$$\pi_{up} = \sum_{i=K+1}^{\infty} (1-\rho)\rho^i = \rho^{K+1} \tag{2}$$

where $\rho = \lambda/\mu$.

Let $\gamma(K, \mu)$ be the rate at which content enters the cache. Due to flow balance, in steady state, $\gamma(K, \mu)$ equals the rate at which content leaves the cache,

$$\gamma(K, \mu) = \mu\rho^{K+1}(1-\rho) = \lambda\rho^K(1-\rho) = \frac{1}{E[B] + E[R]} \tag{3}$$

where the last equality can be easily inferred from Figure 2 (renewal arguments).

$E[B]$ can be calculated from first passage time arguments, that is the time it takes from the system to return to state K (eviction) once it is brought into system (state $(K+1)$). From the M/M/1 model, it can also be calculated by the busy period of an M/G/1 queue, which equals

$$E[B] = 1/(\mu - \lambda). \tag{4}$$

Then, from (4) and (1)

$$\pi_{up} = \frac{1/(\mu-\lambda)}{1/(\mu-\lambda) + E[R]}. \tag{5}$$

Once the content is evicted the mean time for it to return to the cache is given by

$$E[R] = (1 - \pi_{up})/(\pi_{up}(\mu - \lambda)) \tag{6}$$

Given a fixed π_{up}, it is possible to write ρ, μ, $E[R]$ and γ as a function of K,

$$\begin{aligned}
\rho &= \pi_{up}^{1/(K+1)} && (7)\\
\mu &= \lambda\pi_{up}^{-1/(K+1)} && (8)\\
E[R] &= (1-\pi_{up})/(\pi_{up}(\lambda(\pi_{up})^{-1/(K+1)} - \lambda)) && (9)\\
\gamma &= \lambda\pi_{up}((\pi_{up})^{-1/(K+1)} - 1) && (10)
\end{aligned}$$

Note that there is a tradeoff in the choice of K, as increasing the value of K reduces the rate at which content is inserted into the cache (see equation (3)), which in turn reduces the steady state costs to download the content from external sources. The larger the value of K, the smaller the steady state rate at which the content enters and leaves the cache. But increasing the value of K, also increases the mean time for the content to be reinserted into the cache once the content is evicted. The larger the value

of K, the longer the requesters for a given content will have to wait in order to be able to download the content from the cache after it is evicted (see equation (9)).

Let K_{max} be the maximum value allowed for K. Motivated by the tradeoff above, given a fixed value of π_{up} we consider the following optimization problem,

$$\begin{aligned} \min_{K} \quad & \psi(K) = \alpha\gamma + \beta E[R] && (11)\\ \text{such that} \quad & && (12)\\ K \quad & \leq \quad K_{max} && (13)\\ \pi_{up} \quad & = \quad \rho^{K+1} && (14)\\ \rho \quad & = \quad \lambda/(\lambda\pi_{up}^{-1/(K+1)}) && (15) \end{aligned}$$

α and β are used to control the relevance of long term and short term dynamics. The long term dynamics reflect the behavior of the system after a long period of time, during which the rate at which content enters the cache is given by $\mu\rho^{K+1}(1-\rho)$. The short term dynamics reflect the behavior of the system during a shorter period of time, during which one wants to guarantee that the mean time it takes for the content to return to the cache is not too large. We should keep in mind that we choose π_{up} to satisfy cache capacity limitations and system performance.

Substituting (7)-(10) into (11), the objective function is given by,

$$\psi(K) = \alpha\Big[\lambda\pi_{up}((\pi_{up})^{-1/(K+1)} - 1)\Big] + \beta(1-\pi_{up})\frac{1}{\pi_{up}\lambda}\Big[\frac{1}{(\pi_{up})^{-1/(K+1)} - 1}\Big] \quad (16)$$

The value of $E[R]$ must be bounded so as to avoid starvation, as formalized in the propositions below.

Proposition 2.1. *If $\beta = 0$, the optimal strategy consists of setting $K = K_{max}$. If $K_{max} = \infty$, it will take infinite time for the content to be reinserted in the cache once it is evicted for the first time.*

Proof: The objective function is given by

$$\psi(K) = \alpha\lambda\pi_{up}((\pi_{up})^{-1/(K+1)} - 1) \quad (17)$$

The derivative of the expression above with respect to K is

$$\frac{d\psi(K)}{dK} = \alpha\frac{\lambda\log(\pi_{up})\pi_{up}}{(K+1)^2(\pi_{up})^{1/(K+1)}} \quad (18)$$

which is readily verified to be always negative. Therefore, the minimum is reached when $K = K_{max}$. When $K = \infty$ it follows from (8) that μ tends to 0. The mean time for reinsertion of the content in the cache is given by (9) which grows unboundedly as μ tends to 0. □

Proposition 2.2. *If $\beta > 0$ the optimization problem* (11)-(15) *admits a unique minimum $K^\star$.*

The proof is omitted for conciseness.

2.2. Illustrative Example

Next, we consider an illustrative example to show the tradeoff in the choice of K. Let $\pi_{up} = 0.9$ and $\alpha = \beta = 1$. Figure 3 shows how cost first decreases and then increases, as K increases. The optimal is reached for $K = 10$. At that point, we have $E[R] = 0.31$ and $\gamma = 0.32$.

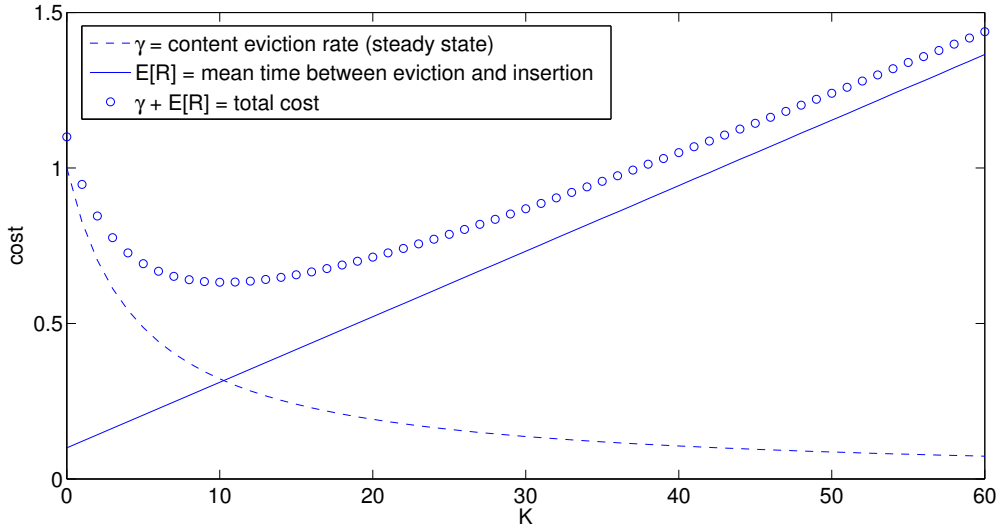


Figura 3. $\pi_{up} = 0.9$, $\alpha = \beta = 1$

Remark 1: Note that in problem (11)-(15) we do not set a hard constraint on $E[R]$. Alternatively, we could add such a constraint, $E[R] \leq R^{\star}$. The solution of the modified problem is either the solution of the problem without the constraint (in case the constraint is inactive) or the value of K which yields $E[R] = R^{\star}$ (in case the constraint is active).

Remark 2: If follows from Markov inequality that the solution of the problem (11)-(15) naturally yields a bound on the probability $P(R > r)$, i.e., $P(R > r) < E[R]/r$. In the numerical example above, when K is optimally set we have that the probability that the the content is not reinserted into cache after 3.1 units of time following an eviction is $P(R > 0.31 \times 10) < 0.1$. In the numerical results we present latter $P(R > r)$ is calculated exactly from the model.

Remark 3: We assume that K can take real values. If K is not an integer, we can always randomize between the two closest integers when deciding whether to store or not the content.

2.3. Reinforced counter with hysteresis

In this section we generalize the reinforced counter to allow for a third control knob K_h as follows. The counter is incremented at each arrival request for a content and is decremented at rate $1/\mu$. In addition, the content is included into cache when the value of the reinforced counter is incremented to $K + 1$, as in previous section. However, content is not removed from cache when the counter value is decremented from $K + 1$ to K. Instead, content remains in cache until the counter reaches the (new) threshold K_h. Figure

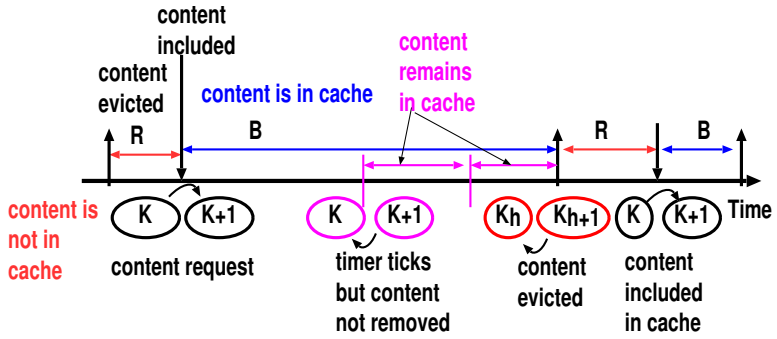


Figura 4. Reinforced counter with hysteresis.

4 illustrates the behavior of the new reinforced counter. From the figure, we observe that the purple intervals correspond to the values of the reinforced counter between K and K_h and the content is in cache. The bottom of Figure 4 shows a trajectory of the reinforced counter.

At first glance, it seems that only the intervals where the content is in cache are affected by this new mechanism (the blue intervals) but not the intervals where content is absent (the red intervals). However, this is not true and both intervals are affected. In what follows, we show how to calculate the expected values of $E[B]$ and $E[R]$ and how the measures of interest are affected by this new mechanism. We also show the advantages of the reinforced counter with hysteresis. For that, we refer to Figure 5 that shows the Markov chain for the hysteresis counter.

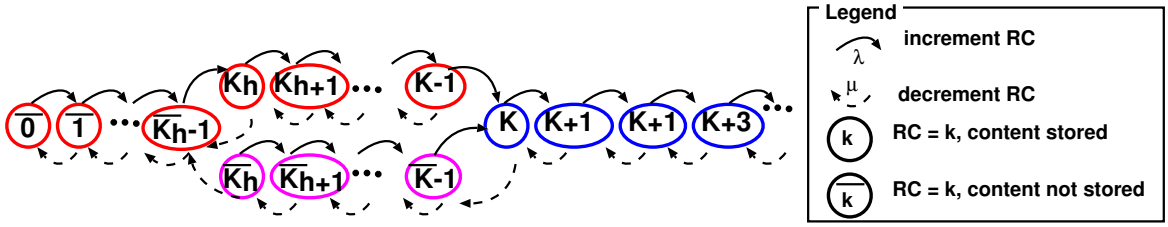


Figura 5. Reinforced counter with hysteresis: state transition diagram.

Let $\nu(i+1) = E[B]$ and $\xi(i+1) = E[R]$ both when $K - K_h = i$. Note that $\nu(1)$ and $\xi(1)$ are the values of $E[B]$ and $E[R]$ for a cache with no hysteresis. $\nu(1)$ is obtained from equations (1), (2) and (9). $\xi(1) = \left(\sum_{j=0}^{K} 1/\rho^j\right) / (\lambda/(\lambda+\mu))$.

Proposition 2.3. *$\nu(i+1)$ and $\xi(i+1)$ can be obtained by the following recursions:*

$$\nu(i) = \tfrac{1}{\mu} + \nu(i-1) + \rho\nu(1) \qquad i \geq 2 \tag{19}$$

$$\xi(i) = \tfrac{1}{\lambda} + \xi(i-1) + \tfrac{1}{\rho}\xi(1) \qquad 2 \leq i \leq K - K_h + 1 \tag{20}$$

Proof: The proof follows from renewal arguments, and is omitted due to space limitations. □

It is important to note that explicit expressions for $\nu(i)$ and $\xi(i)$ can be obtained as a function of λ, μ and K but details are omitted since the recursion above suffices to explain our comments.

Suppose λ and π_{up} are given and we obtain K and μ, for instance from the optimization problem in the previous section. We allow K_h to vary from $K_h = K$ (that is reinforced counter without hysteresis) to $K_h = 0$.

Proposition 2.4. *As K_h decreases, the rate $\gamma(K, \mu)$ at which content enters the cache decreases.*

Proof: From Proposition 2.3, it is not difficult to see that both $E[B]$ and $E[R]$ increase with K_h. Since $\gamma(K, \mu) = 1/(E[B] + E[R])$ (equation (3)) the result follows. □

Proposition 2.4 shows that, from an initial value of π_{up}, if we fix the parameters λ and K, the rate at which content is replaced (both included and removed from cache) decreases by using the hysteresis mechanism, which is good to lower costs as explained in the previous section. However, π_{up} also varies. As a consequence of Proposition 2.3 we can show that π_{up} is reduced. This is not obvious since $E[B]$ increases. But, by adjusting the knob μ, π_{up} can be maintained constant while γ is reduced when the hysteresis schema is used. The proof of this last result is omitted but it follows from Proposition 2.3. The numerical results presented in the Section 2.4 corroborate the claim.

There are additional advantages of the hysteresis mechanism. First note that, in the previous sections, we assumed that file download times are negligible. However, when a user requests for a content it is important that the whole file remains stored at the cache not only until this user finishes downloading but also while other users are downloading the same content from that cache. Hysteresis is helpful to prevent the file from being removed before its download is concluded by all requesters. In Section 2.4 we show that hysteresis increases the probability that a content remains in cache for at least some time t after it is cached. Note that this is an additional (transient) performance measure and it differs from the γ metric (steady state rate). As our numerical results show, by adjusting K_h we can improve both steady state and transient metrics.

Our numerical results also show that hysteresis reduces the coefficient of variation of the time that content resides in the cache (B). This is important for cache capacity planning with multiple contents, since more predictable systems are usually easier to design and control.

2.4. Numerical Results

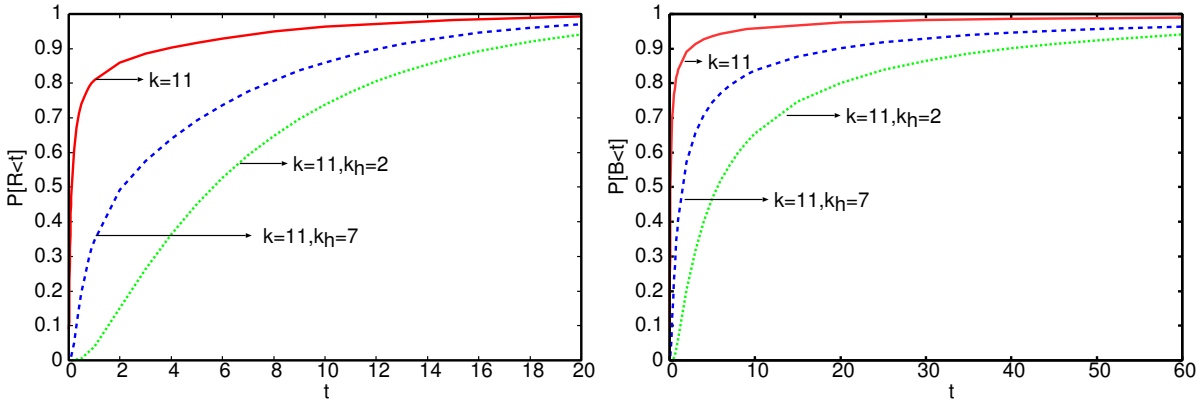
In this section we show some numerical results obtained for the models: the reinforced counter with a single threshold (K) and the reinforced counter with two thresholds (K,K_h). Three performance measures were used to analyze the models: the rate at which content enters the cache (γ), the cumulative distribution of the time the content takes to return to the cache (R) and the cumulative distribution of the time the content remains in the cache after insertion (B).

Three scenarios were evaluated: (a) the reinforced counter has a single threshold $K = 11$, (b) the reinforced counter has two thresholds $K = 11$ and $K_h = 7$, and (c) the reinforced counter has two thresholds $K = 11$ and $K_h = 2$. For each scenario, we consider the same value of π_{up}, λ, and K.

The value of γ for scenario (a) is 0.24, for (b) is 0.07 and for (c) is 0.04. We note that the rate at which content enters or leaves the cache decreases as the value of K_h decreases. This is one of the advantages of introducing a third control knob K_h.

Figures 6(a) and 6(b) show the cumulative distribution of R and B. We note that the reinforced counter with hysteresis allow to control the probability distribution of R and B. In Figure 6(a), consider for example $t = 4$. If we set $K_h = 2$, we have $P[R < 4] = 0.35$ and if $K_h = 7$, then $P[R < 4] = 0.65$. As the value of K_h increases, the probability of R be less than a certain value of t increases. This behavior can also be observed for the distribution of B. On the other hand, if we consider the model with a single threshold we can not control the distribution of R and B. In Figure 6(a), $P[R < 4] = 0.9$.

Another advantage of the reinforced counter with hysteresis, is that the coefficient of variation of R and B, decreases with the value of K_h, which means that the dispersion of the distribution of R and B also decreases. The values obtained for the coefficient of variation of B for each scenario are: (a) 1.6, (b) 1.3 and (c) 1.1.



(a) Cumulative Distribution of R

(b) Cumulative Distribution of B

Figura 6. Cumulative Distribution of the time the content takes to return/remains in the cache.

3. Multiple Caches

In the previous section we argued that we can decouple the cache storage dynamics of each content and study each content in isolation. Roughly, decoupling is a consequence of statistical multiplexing, for content placement policies that limit the amount of time a single content is cached, independently of the other requested contents. This is the case of the *reinforced counter* policies we study. We showed the advantages of these policies and the flexibility they bring to the design of caches.

In this section we address the problem of multiple caches in a network. The main objectives are: (a) to formulate the problem of content placement at a cache network; (b) to show that if we do not use a decoupling policy such as those we study in this paper, the optimal content placement problem is NP hard. This last result emphasizes the importance of employing a cache placement mechanism like the reinforced counters.

3.1. Model and Problem Formulation

Let $\mathcal{F}$ and $\mathcal{C}$ be the set of files and caches in the system. There are $F = |\mathcal{F}|$ files and $C = |\mathcal{C}|$ caches in the system. File f has size t_f, $1 \leq f \leq F$ and cache i has (storage) capacity s_i, $1 \leq i \leq C$. Users issue exogenous requests for files. At each cache i, $1 \leq i \leq C$, exogenous requests for file j arrive at rate λ_{ij}. We assume that cache i has service capacity η_i requests/s. Once a request arrives at a cache,

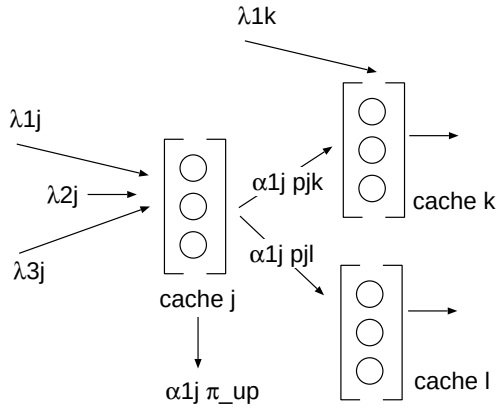


Figura 7. Illustration of cache network

1. **in case of a cache hit**, the content is immediately transferred back to the requester through the data plane. Our model is easily adapted to account for the delay in searching for a file in the cache, but for the sake of presentation conciseness we assume zero searching cost in this paper;
2. **in case of a cache miss**, the request is added to a pool of requests to be serviced. The request is processed and transferred to one of the outgoing links, selected uniformly at random.

Let A_{ij} be a variable that indicates if file j is available at cache i. From the results of previous section, $A_{ij} = 1$ with probability π_{up} for content j at cache i. Therefore, we can compute $P[A_{ij} = 1]$ using the derived results. In addition, we may also study cases in which content is statically placed into caches, i.e., the content in the caches is not replaced. This corresponds, for instance, to a system where the replacements of files from the caches occur at a much coarser granularity than the requests. In this case, $A_{ij} = 1$ if file j is at cache i and 0 otherwise.

Let M_{hi} be the adjacency matrix between caches, i.e., $M_{hi} = 1$ if there is a direct path from cache h to cache i and 0 otherwise.

Let d_h be the outgoing degree of node h. Let p_{hi} be the probability that a request from cache h is routed to cache i. In this paper, we consider random walk routing ([Domingues et al. 2013, Domingues et al. 2014]), i.e., $p_{hi} = 1/|d_h|$ if $M_{hi} > 0$ and 0 otherwise.

Figure 7 illustrates the problem in the case where we have a cache network comprised of 3 nodes, j, k and l, and 3 contents, indexed by 1, 2 and 3. In the Figure, the exogenous arrival rates $\lambda_j^{(1)}, \lambda_j^{(2)}, \ldots$ of contents 1, 2 and 3 to cache j are made explicit. Let $\pi_{up}^{(c)}(i)$ the designed parameter π_{up} in previous sections, but for cache i and content (c). Let $\alpha_j^{(c)}$ represents the total input rate of requests to content (c) arriving at cache j (exogenous arrivals to cache j plus those requests coming from other caches). A fraction $\alpha_j^{(c)}\pi_{up}^{(c)}(j)$ is immediately served, and fractions $\alpha_j^{(c)}(1-\pi_{up}^{(c)}(j))p_{jk}$ and $\alpha_j^{(c)}(1-\pi_{up}^{(c)}(j))p_{jl}$ are transferred to caches k and l, respectively.

In the remainder of this paper, we will assume that requests for files that are cached at i, $1 \leq i \leq C$, are immediately processed at i, and we will be concerned with transfers

that occur due to misses.

$$\alpha_j^{(c)} = \lambda_j^{(c)} + \sum_{k=1,k\neq j}^{C} \alpha_k^{(c)}(1 - \pi_{up}^{(c)}(k))p_{kj} \tag{21}$$

We can compute the mean response time from the expected total number of requests in the system. Using Little's law, and recalling that the processing rate is η_i we have that the average number of requests at cache i is $E[N_i] = \alpha_i/\eta_i$. Then, the expected total number of requests in the system is $E[N] = \sum_{i=1}^{C} E[N_i]$ and the mean response time, $E[T] = E[N]/\Lambda$ where Λ is the sum of the exogenous arrival rates to all caches.

Definition 3.1: The cache network is stable if and only if $\eta_i > \alpha_i$ for all i, $1 \leq i \leq C$. $\diamond$

Until this point, we focused on the use of reinforced counters for content placement. In the remainder of the paper, we show that without reinforced counters a natural statement of optimal content placement yields an NP hard problem.

Definition 3.2: The optimal content placement problem consists of finding the mapping $A : \mathcal{F} \to \mathcal{C}$ such that

1. the cache network is stable
2. $\sum_{f=1}^{F} 1_{A_{if}=1} t_f \leq s_i$, for $i = 1, \ldots F$
3. the arrival rates of requests that require processing at the control plane, $\sum_{i=1}^{C} \alpha_i$, is minimized

$\diamond$

3.2. Static Content Placement in Cache Networks is NP Hard

We show that the problem of static optimal content placement in cache networks is NP hard. This result is interesting because it shows the importance of using a cache replacement policy like those we study in this paper. Recall that, by using the reinforcement policy, we can optimize the counter parameters to satisfy a given probability of finding a content in cache for each content in isolation. These probabilities are in turn obtained to satisfy a given cache capacity constraint, from statistical multiplexing arguments. (the problem is identical to sizing a telephone network and one can use the results of that area.)

First, we consider the case where different files have different sizes. In this case, it is possible to show that the problem is NP hard even if we need to make placement decisions at a single cache. To this aim, we can consider a mapping from KNAPSACK (the proof is omitted due to space constraints). Then, we consider the simpler case wherein all files have the same size. In this case, if we need to make placement decisions at a single cache, the problem can be solved using a greedy strategy. Nevertheless, we show that the problem is still NP hard if we need to make placement decisions in at least two caches.

Theorem 3.1. *The optimal content placement problem is NP hard.*

Proof: We refer to FEASIBLECACHE as the problem of deciding if a given cache network admits a feasible content placement. Note that the problem of finding the optimal placement, i.e. the PLACEMENTPROBLEM, must be harder than FEASIBLECACHE, since solving the optimization problem yields a solution to FEASIBLECACHE. We proceed with a Turing reduction from the PARTITION problem to the FEASIBLECACHE problem. The complete proof is omitted due to space limitations. $\square$

4. Related Work

The literature on caching [Che et al. 2009] and content placement [Presti et al. 2005], and its relations to networking [Neves et al. 2014], database systems and operating systems [Nelson et al. 1988] is vast. Nevertheless, the study of cache networks, which encompass networks where routing and caching decisions are taken together at devices which work as routers and caches is scarce [Rosensweig et al. 2010]. In this work, we present a systematic analysis of cache networks using reinforced counters and indicating their applicability both in the single cache as well as multiple cache settings.

Cache networks play a key role in content centric networks (CCNs) [Jacobson et al. 2009, Koponen et al. 2007]. CCNs are emerging as one of the potential architectures for the future Internet. In CCNs, content rather than hosts is addressed. The *glue* that binds CCNs are the content chunks rather than the IP packets [Jacobson et al. 2009].

Using caches to reduce publishing costs can be helpful for publishers that cannot afford staying online all the time, or need to limit the bandwidth consumed to replicate content. With caches, content can *persist* in the network even in the absence of a publisher [Koponen et al. 2007]. In case of intermittent publishers, content availability can only be guaranteed in case replicas of the content are placed in some of the caches.

Approximate and exact analysis of single caches and cache networks has been studied for decades [Berger et al. 2014, Bianchi et al. 2013, Fofack et al. 2012]. Approximate analysis is usually carried out using mean field approaches [Bianchi et al. 2013] or other asymptotic techniques [Jelenković 1999], whereas exact analysis is performed accounting for Markov chain properties [Gast and Houdt 2015] or assuming uncoupled content dynamics [Fofack et al. 2012]. In this paper, we consider reinforced counters as the placement algorithms, which are at the same time amenable to analytical study and of practical interest in the context of DNS systems or the novel Amazon ElastiCache [Amazon 2014, Berger et al. 2014]. Cache policies are traditionally devised to avoid staleness of content and/or improve system efficiency. The analysis presented in this paper shows that reinforced counters can be used to target these two goals.

5. Conclusion and Future Work

In this paper we have studied the problem of scalable and efficient content placement in cache networks. We analyzed an opportunistic caching policy using reinforced counters [Domingues et al. 2013] as an efficient mechanism for content placement and proposed an extension to the basic scheme (reinforced counters with hysteresis). Using reinforced counters, we indicated how to tune simple knobs in order to reach the optimal placement. We then showed some of the properties of the mechanism and the optimal content placement problem. Finally, using the queueing-network model we propose for a cache network, we showed how to calculate the overall expected delay for a request to obtain a content. In addition, we showed that obtaining the optimal solution without reinforced counters is an NP hard problem.

This work is a first step towards the characterization of efficient, scalable and tractable content placement in cache networks. Future work consists of studying how to distributedly tune the caches, allowing the presence of custodians that store permanent copies of the files. We also aim at simulating the proposed solutions under real topologies [Jannibelli 2014].

Referências

Amazon (2014). Amazon elasticache. `aws.amazon.com/elasticache/`.

Berger, D. S., Gland, P., Singla, S., and Ciucu, F. (2014). Exact analysis of ttl cache networks: The case of caching policies driven by stopping times. *arXiv preprint arXiv:1402.5987*.

Bianchi, G., Melazzi, N. B., Caponi, A., and Detti, A. (2013). A general, tractable and accurate model for a cascade of caches. *arXiv preprint arXiv:1309.0718*.

Che, H., Tung, Y., and Wang, Z. (2009). Hierarchical web caching systems: Modeling, design and experimental results. In *JSAC*.

Domingues, G., de Souza e Silva, E., Leão, R. M. M., and Menasché, D. S. (2013). Enabling information centric networks through opportunistic search, routing and caching. In *SBRC*, pages 135–148.

Domingues, G., de Souza e Silva, E., Leão, R. M. M., and Menasché, D. S. (2014). A name resolution assisted icn design, supported by opportunistic search, routing and caching policies. In *Workshop em Desempenho de Sistemas Computacionais e de Comunicação (Wperformance)*.

Fofack, N. C., Nain, P., Neglia, G., and Towsley, D. (2012). Analysis of ttl-based cache networks. In *VALUETOOLS*, pages 1–10. IEEE.

Gast, N. and Houdt, B. V. (2015). Transient and steady-state regime of a family of list-based cache replacement algorithms. In *SIGMETRICS*.

Jacobson, V., Smetters, D. K., Briggs, N. H., Thornton, J. D., Plass, M. F., and Braynard, R. L. (2009). Networking named content. In *CONEXT*.

Jannibelli, D. V. (2014). Simulador para um modelo de busca oportunística de conteúdos em redes orientadas a informação. Master's thesis, COPPE UFRJ, Rio de Janeiro.

Jelenković, P. R. (1999). Asymptotic approximation of the move-to-front search cost distribution and least-recently used caching fault probabilities. *Annals of Applied Probability*, pages 430–464.

Koponen, T., Chawla, M., Chun, B., Ermolinskiy, A., Kim, K., Shenker, S., and Stoica, I. (2007). A data-oriented (and beyond) network architecture. In *SIGCOMM*.

Nelson, M. N., Welch, B. B., and Ousterhout, J. K. (1988). Caching in the sprite network file system. *ACM Transactions on Computer Systems (TOCS)*, 6(1):134–154.

Neves, T., Ochi, L. S., and Albuquerque, C. (2014). A new hybrid heuristic for replica placement and request distribution in content distribution networks. *Optimization Letters*, pages 1–16.

Presti, F. L., Bartolini, N., and Petrioli, C. (2005). Redirection in content delivery networks. In *ICC*.

Rosensweig, E. J., Kurose, J., and Towsley, D. (2010). Approximate models for general cache networks. In *INFOCOM*.

Zhang, L. and Carofiglio, G. (2012). 1st workshop on emerging design choices in name-oriented networking. In *INFOCOM*.